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Bending Response of Doubly Curved Laminated Composite Shells using Hybrid Refined Models

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Abstract. This study presents a static analysis of laminated composite doubly curved shells using a refined kinematic model with polynomial and non-polynomial functions. In particular Maclaurin, trigonometric, exponential and zig-zag functions are employed. Refined models are based on the Equivalent Single Layer theories and obtained by using Carrera Unified formulation. The shell model is subjected to different mechanical loading such as bi-sinusoidal, uniform and point load. The governing equations are derived from the principle of virtual displacement and solved via Navier-Type closed form solutions. The results are compared with Layer-wise and higher-order shear deformation solutions available in the literature. It is shown that refined models with non-polynomial terms are capable of accurately predicting the through-the-thickness displacements and stress distributions with a low computational effort.

1. Introduction

Multilayered shell structures are utilized in many engineering fields such as aerospace, naval and automotive due to their superior characteristics. Shells are curved structures which have excellent load-carrying capacity in comparison to plates. Moreover, composite structures provide high strength, high stiffness-weight ratio and remarkable fatigue resistance compared to metal structures. Nevertheless, composite structures show local problems such as delamination that needs to be investigated with accurate and cost-efficient simulations. Consequently, reliable and efficient computational models for multilayered structures are still an important research topic.

Shell theories have been formulated for over 70 years. The First order shear deformation theory (FSDT) was proposed by Hildebrand et al. [1] in order to include the effect of transverse shear stress. The FSDT depends on a shear correction factor which is difficult to estimate for composite shells. Consequently, Higher Order Shear Deformation theories (HSDTs) [2, 3] were reported on the literature. HSDTs include additional terms on the kinematic model in order to accurately predict laminated composites displacements and stress distributions.

A generalization of HSDTs is the so-called Unified Formulations (UFs) [4]. UFs permits to write governing equations of different models within a unique computer program. The fact that in some HSDTs the number and type of shear strain functions are considered constant is a clear disadvantage in comparison to UFs. An excellent review on plates and shells theories for laminated and sandwich structures comparing HSDTs and UFs are presented by Caliri et al. [5].

Shell multilayered models can be implemented in two different approaches: Equivalent Single Layer (ESL) and Layer-wise (LW) [4]. An ESL model considers multilayered structures as a single equivalent layer. A LW model maintains each lamina separately; therefore, accurate displacements and stress distributions are obtained in comparison to ESL models. However, the high computational cost of LW models makes it difficult to implement in engineering applications. Consequently, robust ESL models with adequate precision are preferred.

The developed formulation is based on Carrera Unified formulation (CUF) with the inclusion of non-polynomial expansion function to study for the first time shell panels. According to CUF, the displacement field of the shell can be defined as an arbitrary expansion of the thickness coordinates. The governing equations are the so-called fundamental nuclei whose form does not depend on either the expansion order or the choices made for the shear strain shape functions. The highly coupled differential equations are solved via Navier-Type solutions and several static problem of shells are analyzed under different loading.

A significant number of investigations have been presented regarding shell behavior. Tornabene et al. [6] presented a stress recovery method with CUF and the differential geometry tool to obtain quasi-3D results for doubly curved anisotropic shells and panels. Dozio [7] used a space-state approach in conjunction to Levy's method for solving free vibration problem of spherical and cylindrical panels by applying a hierarchical formulation. Brischetto [8] proposed a three-dimensional elasticity solution for the static and free vibration analyses of simply supported spherical shells based on the exponential matrix method. Viola et al. [9] presented a new HSDT for free vibration and static analysis of shells. Tornabene et al. [10] applied CUF for the free vibration of doubly curved laminated shells and panels, the highly coupled differential equation were solved numerically applying radial basis functions. Cinefra and Valvano [11] combined CUF and mixed interpolation of tensorial component for solving different problems of anisotropic doubly curved shell.

The paper proposes different refined hybrid models (HRM) including non-polynomial functions for the static analysis of simply supported laminated composite doubly curved shells. The formulation includes polynomial, trigonometric, exponential and zig-zag functions. The kinematic models are adapted from the HRMs presented by Yarasca et al. [12, 13]. This paper aims to study the capability of this formulation to model shell structures.

2. Shell formulation and close form solution

Let $\{\alpha,\beta\}$ be a set of orthogonal curvilinear coordinates of a doubly curved shell. The thickness is denoted as "h", the length as "a" and the width as "b". The radii of curvature "R_α" and "R_β" are considered as constant along the midsurface domain Ω_k. The integer k denotes the layer number from the shell-bottom.

The presented paper studies the influence of different trigonometrical, exponential and zig-zag shear strain shape functions in ESL models. The displacement field contains 54 terms: 15 Maclaurin, 3 zig-zag terms, 24 trigonometrical terms and 12 exponentials terms. The non-polynomial functions were selected by Filippi et al. [14] for plates and were used by Yarasca et al. [13]. For example, the expansion along β axis is denoted as:

$$u_\beta = u_{\beta 0} + z u_{\beta 1} + z^2 u_{\beta 2} + z^3 u_{\beta 3} + z^4 u_{\beta 4} + (-1)^k \zeta_k u_{\beta 5} + \sin\left(\frac{\pi z}{h}\right) u_{\beta 6} + \sin\left(\frac{2\pi z}{h}\right) u_{\beta 7}$$
$$+ \sin\left(\frac{3\pi z}{h}\right) u_{\beta 8} + \sin\left(\frac{4\pi z}{h}\right) u_{\beta 9} + \cos\left(\frac{\pi z}{h}\right) u_{\beta 10} + \cos\left(\frac{2\pi z}{h}\right) u_{\beta 11} + \cos\left(\frac{3\pi z}{h}\right) u_{\beta 12}]$$
$$+ \cos\left(\frac{4\pi z}{h}\right) u_{\beta 13} + e^{\frac{z}{h}} u_{\beta 14} + e^{\frac{2z}{h}} u_{\beta 15} + e^{\frac{3z}{h}} u_{\beta 16} + + e^{\frac{4z}{h}} u_{\beta 17} \quad (1)$$

Refined models are defined as N hybrid refined models (N-HRM), where the N notation is the number of the variables in the model. The principle of virtual displacement for multilayered doubly curved shells is given by:

$$\sum_{k=1}^{N_l} \int_{\Omega_k} \int_{A_k} \left\{ \delta\varepsilon_p^{k^T} \sigma_p^k + \delta\varepsilon_n^{k^T} \sigma_n^k \right\} H_\alpha{}^k H_\beta{}^k \, \mathrm{d}\Omega_k \mathrm{d}z = \sum_{k=1}^{N_l} \int_{\Omega_k} \int_{A_k} \{\delta u^k p^k\} H_\alpha{}^k H_\beta{}^k \mathrm{d}\Omega_k \mathrm{d}z \quad (2)$$

where Ω_k and A_k represents the integration domains in plane and the thickness direction, respectively. p^k is the mechanical load applied at a certain k layer.

The numerical results were obtained via the Navier closed-form solution for simply supported orthotropic panels. The displacement expressed as a summation of harmonics:

$$u_{\alpha_s}^k = \sum_{m,n} U_{\alpha_s}^k \cos\left(\frac{\pi m}{a}\alpha\right)\sin\left(\frac{\pi n}{b}\beta\right)$$
$$u_{\beta_s}^k = \sum_{m,n} U_{\beta_s}^k \sin\left(\frac{\pi m}{a}\alpha\right)\cos\left(\frac{\pi n}{b}\beta\right) \tag{3}$$
$$u_{z_s}^k = \sum_{m,n} U_{z_s}^k \sin\left(\frac{\pi m}{a}\alpha\right)\sin\left(\frac{\pi n}{b}\beta\right)$$

where "m" and "n" are the number of waves and $U_{\alpha_s}^k$, $U_{\beta_s}^k$ and $U_{z_s}^k$ are related to the amplitudes of the Fourier series displacement.

A 2D elastic theory for shells with constant radii of curvature is solved. This paper presented a simplified model of the 3D elasticity theories with the assumptions of the displacement field shown in equation (1). The presented refined model can capture the real behavior of shell structures for displacement and in-plane stresses. However, a refinement is needed for transverse shear and normal stresses. The 3D elasticity are valid for solving problems of shells with simply supported boundary conditions. The shear stresses and their derivatives are replaced into the 3D elasticity equations in order to calculate the out-plane stresses. The 3D elasticity equations are reported by Brischetto [8] and given by:

$$H_\beta^k \frac{\partial \sigma_{\alpha\alpha}^k}{\partial \alpha} + H_\alpha^k \frac{\partial \sigma_{\alpha\beta}^k}{\partial \beta} + H_\alpha^k H_\beta^k \frac{\partial \tau_{\alpha z}^k}{\partial z} + \left(\frac{2H_\beta^k}{R_\alpha^k} + \frac{H_\alpha^k}{R_\beta^k}\right)\tau_{\alpha z}^k = 0$$
$$H_\beta^k \frac{\partial \sigma_{\alpha\beta}^k}{\partial \alpha} + H_\alpha^k \frac{\partial \sigma_{\beta z}^k}{\partial \beta} + H_\alpha^k H_\beta^k \frac{\partial \tau_{\beta z}^k}{\partial z} + \left(\frac{2H_\alpha^k}{R_\beta^k} + \frac{H_\beta^k}{R_\alpha^k}\right)\tau_{\beta z}^k = 0 \tag{4}$$
$$H_\beta^k \frac{\partial \tau_{\alpha z}^k}{\partial \alpha} + H_\alpha^k \frac{\partial \tau_{\beta z}^k}{\partial \beta} + H_\alpha^k H_\beta^k \frac{\partial \sigma_{zz}^k}{\partial z} - \frac{H_\beta^k}{R_\alpha^k}\sigma_{\alpha\alpha}^k - \frac{H_\alpha^k}{R_\beta^k}\sigma_{\beta\beta}^k + \left(\frac{H_\beta^k}{R_\alpha^k} + \frac{H_\alpha^k}{R_\beta^k}\right)\sigma_{zz}^k = 0$$

The formulation must satisfy the zero shear strain condition at the bottom and the top of the shell structure and interlaminar continuity. Equations (4) could not be solved analytically, therefore, an approximate solution using Taylor series is adopted. These equations are valid for doubly curved shells with constant radii of curvature.

3. Results and discussions

In this section, some results of the static analysis of composite doubly curved shell panels are presented. The boundary conditions are considered as simply supported. The geometry data is considered as: $a = b = 1$ and $R_\alpha = R_\beta = R$. Different side-to-thickness (*a/h*) and curvature-radius-to-side (*R/a*) ratios are considered as parameters. The first case is related to a laminated doubly-curvature panel with the stacking sequence 0°/90°/0° subjected to bi-sinusoidal loading. Some results are compared with different HSDTs [2, 6, 15, 16] available in the literature. The second case is a benchmark solution of two laminae. The original problem was presented by Demasi [4] for plates and it is used to analyze doubly curved panels in this paper.

The mechanical properties of each layer for the first case were reported by Reddy and Liu [2]:

$$E_1 = 25E_2, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, \upsilon_{12} = \nu_{13} = 0.25 \tag{5}$$

In this paper, the transverse displacement $\bar{w}$ and transverse normal stress $\bar{\sigma}_{zz}$ is evaluated at (a/2, b/2, 0). The normal stress $\sigma_{\alpha\alpha}$ is evaluated at (a/2, b/2, h/2). The transverse shear stress $\tau_{\alpha z}$ and $\tau_{\beta z}$ are computed at (0, b/2, 0) and (a/2, 0, 0), respectively.

Figure 1 shows dimensionless stresses $\bar{\sigma}_{\alpha\alpha}$, $\bar{\sigma}_{\alpha z}$, $\bar{\sigma}_{\beta z}$ and $\bar{\sigma}_{zz}$ distributions through the thickness. The results demonstrated a good accuracy overall the thickness distributions with respect to the LD4. Figure 2 shows the relation between the dimensionless $\bar{u}_z$ and the inverse of curvature-radius-to-size *(a/R)* ratio for a moderate thick shell *a/h* = 10. The displacements are compared with other formulations available in the literature such as the sinusoidal shear deformation theory proposed by Ferreira et al. [16], the third order shear deformation proposed by Reddy and Liu [2], etc.

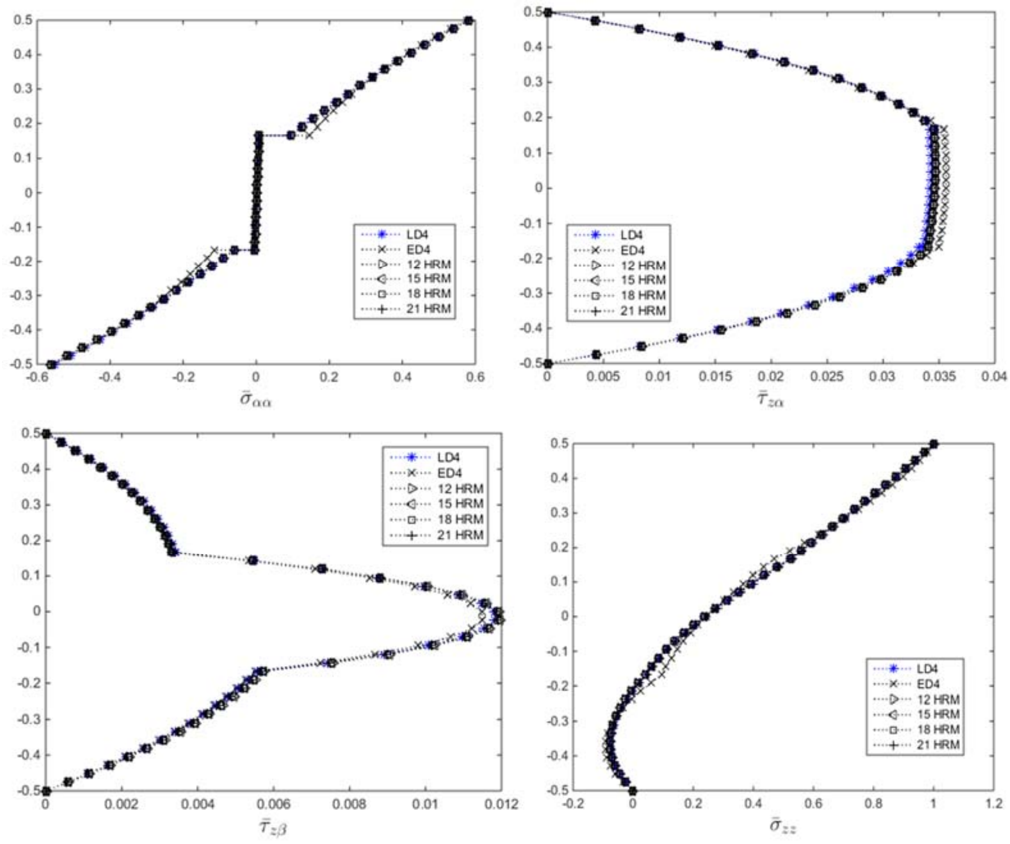

Figure 1. Dimensionless stress distributions for 0°/90°/0°, a/h = 10, R/a = 5.

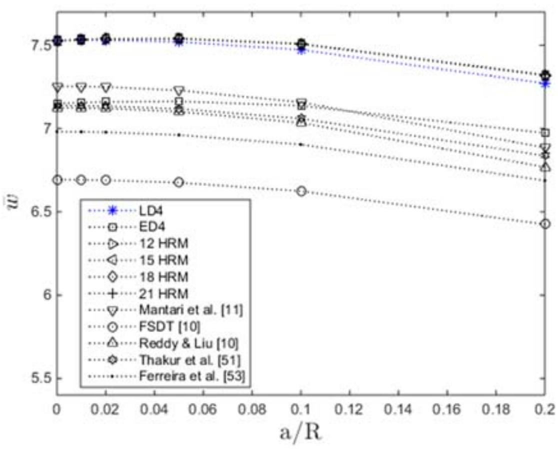

Figure 2. Dimensionless transverse displacement $\bar{u}_z$ versus a/R ratio for 0°/90°/0°, a/h = 10.

The second case problem was studied by Demasi [4] for plate configuration. In this paper, it is extended to doubly-curvature panel. The mechanical properties of layers are: $E_{11} = 25E_{22}$, $E_{11} = 25E_{33}$, $E_{11} = 50\ G_{12}$, $E_{11} = 50G_{13}$, $E_{11} = 125G_{23}$, $\varphi = 0$ for the bottom and $E_{11} = 25E_{22}$, $E_{11} = 2.5E_{33}$, $E_{11} = 50\ G_{12}$, $E_{11} = 50G_{13}$, $E_{11} = 125G_{23}$, $\varphi = 90$ for the top. The Poisson ratio is considered as: $v_{12} = v_{13} = v_{21} = 0.25$ for both layers. The thickness ratio is considered as *a/h = 4*. Different curvature-radius-to-size ratios are investigated. Stresses $\sigma_{\alpha\alpha}$, $\tau_{\alpha z}$ and $\tau_{\beta z}$ are calculated at (a/2,b/2,h/4), (0,b/2,h/4) and (a/2,0,h/4), respectively. Figure 3 shows the through thickness distributions of these the stresses $\bar{\sigma}_{\alpha\alpha}$, $\bar{\tau}_{\alpha z}$, $\bar{\tau}_{\beta z}$ and $\bar{\sigma}_{zz}$. Good agreements in the results are obtained overall the thickness distribution.

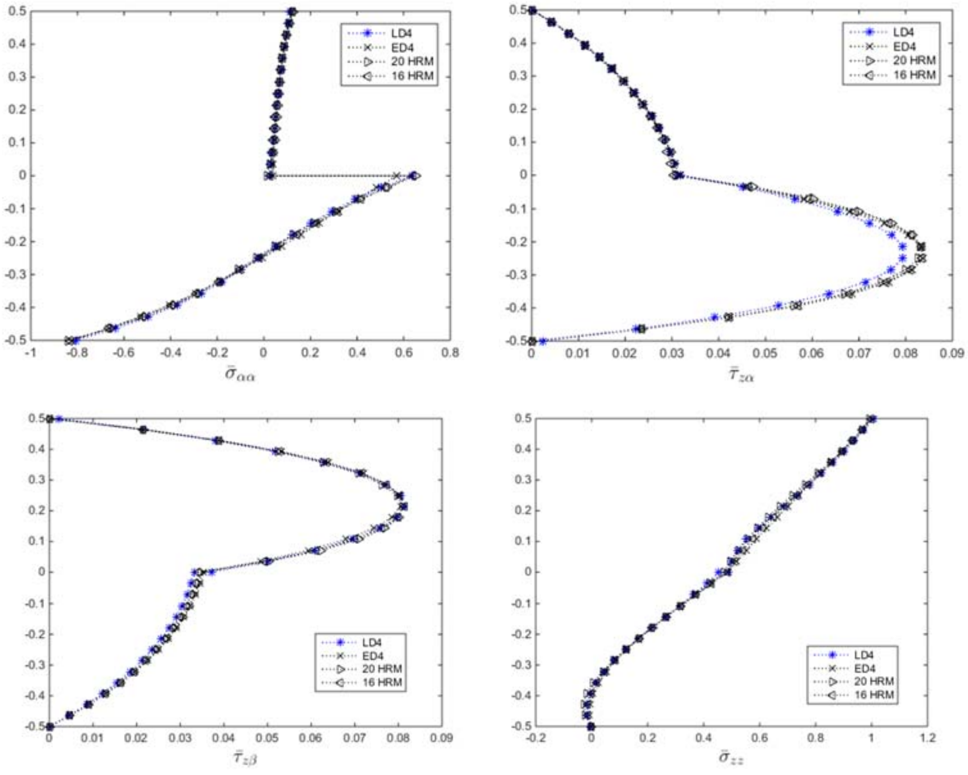

Figure 3. Distribution of dimensionless stresses for Demasi benchmark.

4. Conclusions

This paper presented an analytical solution for the static analysis of doubly curved shells with constant curvature. The shear strain shape functions were considered as hybrid with different combinations of trigonometric, exponential, zig-zag and Maclaurin polynomials thickness expansions. Simply supported shells subjected to bi-sinusoidal loading are analyzed by using the principle of virtual displacement and solved via Navier-closed form solutions. Stresses for different thickness ratios and curvature-radius-to-size are calculated. This work investigated how different types of loadings and the effect of the curvature affected the HRM for shells. The following conclusions are obtained:

(1) Trigonometric functions showed better results over exponential functions for 0/90/0 lamination.

(2) The used of exponential, trigonometric and zig-zag shear functions in the kinematic models are mandatory for the adequate simulation of complex laminations as the one proposed by Demasi.

References

[1] Hildebrand F, Reissner E and Thomas G 1949 *Notes on the foundations of theory of small displacements of orthotropic shells* NACA technical notes No. 1883.
[2] Reddy J N and Liu C 1985 *Int. J. Eng. Sci.* **23** 319-30.
[3] Mantari J and Guedes Soares C 2012 *Comp. Struct.* **94** 2640-56.
[4] Demasi L 2009 *Comp.* **88** 1-16.
[5] Caliri M *et al* 2006 *Comp. Struct.* **156** 63-77.
[6] Tornabene F *et al* 2014 *Comp. Struct.* **107** 675-97.
[7] Dozio L 2016 *Comp. Part B* **98** 97-107.
[8] Brischetto S 2017 *Comp. Part B* **119** 230-52.
[9] Viola E *et al* 2013 *Comp. Struct.* **101** 59-93.
[10] Tornabene F *et al* 2013 *Comp. Part B* **55** 642-59.
[11] Cinefra M and Valvano S 2016 *Mech. Adv. Mater. Struct.* **23**.
[12] Yarasca J *et al* 2017 *Comp. Struct.* **161** 362-83.
[13] Yarasca J *et al* 2017 *Comp. Struct.* **176** 860-76.

[14] Filii M *et al* 2016 *Comp. Struct.* **150** 103-14.
[15] Sandipan N 2017 *Acta Mech.* **228** 69-87.
[16] Ferreira A J M 2011 *Comp.: Part B* **42** 1276-84.