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Higher-order non-local finite element bending analysis of functionally graded beams

TRABAJO DE INVESTIGACIÓN

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AUTOR(ES)

Garbin Turpaud, Fernando (0000-0001-8201-6564)

Lévano Pachas, Ángel Alfredo (0000-0003-0285-8003)

ASESOR

Arciniega Alemán, Román Augusto (0000-0002-4284-4389)

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DEDICATORIA

A nuestros padres, hermanos, y a la memoria de aquellos que amamos y ya no están con nosotros.

AGRADECIMIENTOS

Si tuviéramos que mencionar a todas las personas que alguna vez nos apoyaron, creyeron en nosotros y tuvieron significancia en nuestras vidas, esta tesis solo serían nombres.

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Un agradecimiento especial a nuestra familia y amigos, quienes siempre nos apoyaron para que estudiemos una carrera y que diésemos lo mejor de nosotros.

RESUMEN

La teoría de vigas de Timoshenko TBT y una teoría de alto orden IFSDT son formuladas utilizando las ecuaciones constitutivas no locales de Eringen. Se utilizaron ecuaciones constitutivas en 3D en el modelo IFSDT. Se utilizó una variación del material con el uso de materiales funcionalmente graduados a lo largo del peralte de una viga de sección rectangular. El principio de trabajos virtuales utilizado y ejemplos numéricos fueron presentados para comparar ambas teorías de vigas.

Palabras clave: Teorías no locales; Vigas funcionalmente graduadas; Método de los elementos Finitos; Modelo no local de Eringen; Condiciones de borde

Higher-order non-local finite element bending analysis of functionally graded beams

ABSTRACT

Abstract-. Timoshenko Beam Theory (TBT) and an Improved First Shear Deformation Theory (IFSDT) are reformulated using Eringen's non-local constitutive equations. The use of 3D constitutive equation is presented in IFSDT. A material variation is made by the introduction of FGM power law in the elasticity modulus through the height of a rectangular section beam. The virtual work statement and numerical results are presented in order to compare both beam theories.

Keywords: Non-local theories, FGM Beams, Finite Element Method, Eringen's non-local model, Boundary conditions

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I. INTRODUCTION

Classical continuum mechanics theories commonly have a local approach, which assumes that the stress at a point just depends on the strain on that same point. These theories are true enough for numerous case of study. However, in other cases, such as lattice dispersion of waves, waves' propagation (Eringen, 1983), crack propagation, dislocations and surface tension fluids, they are no longer sufficient (Srinivasa & Khodabakhshi, 2016). Eringen (Eringen, 1972) proposed a nonlocal constitutive behavior. This behavior implies that the stress of a point is not only dependent on the strain of that point, but also on the strain of rest of the continuum. Nonlocal theories allowed further investigation in fields such as nanomaterials.

A comparison between different beam theories (varying the displacement fields) are presented by (Aydogdu, 2009; Eltaher, Khairy, Sadoun, & Omar, 2014; J. N. Reddy, 2007) using linear approaches and nonlocal constitutive equations to analyze bending, buckling and vibration. This includes Timoshenko Beam Theory and other higher order theories. In the present study the Improved First Shear Deformation Theory (IFSDT), proposed by (Arciniega, 2005) using seven independent parameters in order to solve a finite element model of shell structures is approximated into beams such as (Soncco & Jorge, 2019) and (Soncco, Jorge, & Arciniega, 2019) did by analyzing nonlinear bending and post-buckling analysis of Functionally Graded beams. However, in the present study nonlocal constitutive equations are introduced and then compared with previous work only using a linear approximation.

Finite element solutions, within nonlocal beam theories (Timoshenko and other higher order theories), have already been proposed by (Alshorbagy, Eltaher, & Mahmoud, 2013; Eltaher et al., 2014; J. N. Reddy, 2010; J. N. Reddy & El-Borgi, 2014). However, finite element solutions of IFSDT in beams, proposed by (Soncco & Jorge, 2019) and (Soncco et al., 2019) does not contemplate nonlocal constitutive equations.

In the present investigation, a non-local linear finite element model is evaluated and compared with the literature in order to evaluate a non-local Timoshenko FEM model and a non-local IFSDT FEM model.

II. METHOD

2.1 Non-local differential model

As is presented by Karlicic (Karličić, Murmu, Adhikari, & McCarthy, 2015), according to Eringen, the stress at a point does not only depends on the strain at that same point, but also on the strains at the rest of the continuum body. Eringen describes this phenomenon based on the atomic theory of lattice dynamics and experimental observations. The non-local stress tensor $\boldsymbol{\sigma_{ij}^{nl}}$ (also seen as an "absolute" stress) at point $\boldsymbol{x}$ is defined as:

$$\sigma_{ij}^{nl} = \int_V K(|x' - x|, \emptyset)\sigma_{ij}(x')dV(x') \tag{1}$$

Where $\boldsymbol{\sigma_{ij}(x')}$ is the local stress tensor at $\boldsymbol{x'}$ and the Kernel function $K(|x' - x|, \emptyset)$ is declared as the non-local modulus that adds the stress effect caused by $\boldsymbol{x'}$ with reference to $\boldsymbol{x}$, in which $|x' - x|$ represents the Euclidian distance between $\boldsymbol{x'}$ and $\boldsymbol{x}$. Whereas $\emptyset$ is a constant, being $\emptyset = e_0 \cdot a / l$, where e_0 is a material dependent constant, and a and l are internal and external characteristic lengths respectively.

However, the local stress $\boldsymbol{\sigma_{ij}}$ at $\boldsymbol{x}$ is defined, using elastic constitutive equations, by the generalized Hooke's law as:

$$\sigma_{ij}(x) = C_{ijkl}(x) : \varepsilon_{kl}(x) \tag{2}$$

However, although the non-local stress-strain relation is based on an integral constitutive equation, Eringen also proposed an equivalent differential model:

$$(1 - \mu\nabla^2).\sigma_{ij}^{nl} = \sigma_{ij} = C_{ijkl}.\varepsilon_{kl} \tag{3}$$

Where $\mu = \emptyset^2 l^2$

Governing Equations for beam theories

2.2 Timoshenko Beam Theory

The Timoshenko Beam Theory (TBT) has the following displacement field:

$$
\begin{aligned}
v_1 &= u(x_1) + \phi_1(x_1).x_3 \\
v_2 &= 0 \\
v_3 &= w(x_1)
\end{aligned}
\tag{4}
$$

Where $\boldsymbol{u(x_1)}$ and $\boldsymbol{w(x_1)}$ are axial and transverse displacement respectively and $\boldsymbol{\phi_1(x_1)}$ is the rotation of the cross-section.

2.2.1 Local Stress Resultants

From the Green-Lagrange strain tensor:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \left[\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right]\alpha\right) \tag{5}$$

The present study will use a linear approximation ($\alpha = 0$). Replacing the displacement field we obtain:

$$\varepsilon_{11} = \frac{\mathrm{d}v_1}{\mathrm{dx}_1} = \frac{\mathrm{du}}{\mathrm{dx}_1} + \frac{\mathrm{d}\phi_1}{\mathrm{dx}_1}.\mathrm{x}_3 \tag{6}$$

$$\varepsilon_{13} = \frac{1}{2}\left(\frac{\mathrm{d}v_1}{\mathrm{dx}_3} + \frac{\mathrm{d}v_3}{\mathrm{dx}_1}\right) = \frac{1}{2}\left(\phi_1 + \frac{\mathrm{dw}}{\mathrm{dx}_1}\right) \tag{7}$$

From the Principle of Virtual Works and replacing Timoshenko's Plane Stress Constitutive equations we obtain the following stress resultants:

$$
\begin{aligned}
\mathrm{N}_{11} &= E.\mathrm{A}.\varepsilon_{11}^{(0)} \rightarrow \mathrm{N}_{11} = \mathrm{EA}\frac{\mathrm{du}}{\mathrm{dx}_1} \\
\mathrm{M}_{11} &= E.\mathrm{I}.\varepsilon_{11}^{(1)} \rightarrow \mathrm{M}_{11} = EI\frac{\mathrm{d}\phi_1}{\mathrm{dx}_1} \\
\mathrm{N}_{13} &= \frac{E}{2}\left(2\varepsilon_{13}^{(0)}\right) \rightarrow \mathrm{N}_{13} = \mathrm{GAK_S}\left(\phi_1 + \frac{\mathrm{dw}}{\mathrm{dx}_1}\right)
\end{aligned}
\tag{8}
$$

By replacing at the Virtual Work Statement we obtain:

$$\int_{x1}\left(f_1.\delta\mathrm{u} + N_{11}\frac{\mathrm{d}\delta\mathrm{u}}{\mathrm{dx}_1}\right) + \left(\mathrm{N}_{13}\delta\phi_1 + \mathrm{M}_{11}\frac{\mathrm{d}\delta\phi_1}{\mathrm{dx}_1}\right) + \left(f_3.\delta\mathrm{w} + \mathrm{N}_{13}\frac{\mathrm{d}\delta\mathrm{w}}{\mathrm{dx}_1}\right)\mathrm{dx}_1 \tag{9}$$

2.2.2 Non-Local Stress Resultants

From Euler-Lagrange equations we obtain that

$$\frac{dN_{11}^{nl}}{dx} + f_1 = 0 \; ;$$
$$\frac{dM_{11}^{nl}}{dx} - N_{13}^{nl} = 0 \; ; \qquad (10)$$
$$\frac{dN_{13}^{nl}}{dx} + f_3 = 0$$

By replacing the Eringen's non-local constitutive equation

$$N_{11}^{nl} - \mu.\frac{d^2 N_{11}^{nl}}{dx^2} = N_{11} \; ;$$
$$M_{11}^{nl} - \mu.\frac{d^2 M_{11}^{nl}}{dx^2} = M_{11} \; ; \qquad (11)$$
$$N_{13}^{nl} - \mu.\frac{d^2 N_{13}^{nl}}{dx^2} = N_{13}$$

Clearing the non-local stress resultants we obtain:

$$N_{11}^{nl} = N_{11} - \mu.\frac{df_1}{dx} \; ;$$
$$M_{11}^{nl} = M_{11} - \mu.f_3 \; ; \qquad (12)$$
$$N_{13}^{nl} = N_{13} - \mu.\frac{df_3}{dx}$$

f_1 and f_3 are the distributed axial and transverse force respectively. By substituting the non-local (actual) stress resultants in the Virtual Work Statement we obtain:

$$\int_{x_1} EA\frac{du}{dx_1}\frac{d\delta u}{dx_1} - \mu\frac{df_1}{dx_1}\frac{d\delta u}{dx_1} - f_1\delta u + EI\frac{df_1}{dx_1}\frac{d\delta f_1}{dx_1} - \mu f_3\frac{d\delta f_1}{dx_1} + GAK_S\left(f_1 + \frac{dw}{dx_1}\right)\left(\delta f_1 + \frac{d\delta w}{dx_1}\right) - \mu\frac{df_3}{dx}\left(\delta f_1 + \frac{d\delta w}{dx_1}\right) - f_3.\delta w\, dx_1 \qquad (13)$$

2.2.3 Finite Element Formulation

Re-arranging the Virtual Work Statement obtained in section 3.1.3 we gathered the following equations:

$$\int_{x_1} EA\frac{du}{dx_1}\frac{d\delta u}{dx_1} - \mu.\frac{df_1}{dx_1}\frac{d\delta u}{dx_1} - f_1\delta u \, dx_1 = 0 \qquad (14)$$

$$\int_{x_1} EI\frac{df_1}{dx_1}\frac{d\delta f_1}{dx_1} - \mu.f_3\frac{d\delta f_1}{dx_1} + GAK_S\left(f_1 + \frac{dw}{dx_1}\right)\delta f_1 - \mu.\frac{df_3}{dx}\delta f_1 dx_1 = 0 \qquad (15)$$

$$\int_{x_1} GAK_S\left(f_1 + \frac{dw}{dx_1}\right)\frac{d\delta w}{dx_1} - \mu.\frac{df_3}{dx}\frac{d\delta w}{dx_1} - f_3.\delta w \, dx_1 = 0 \qquad (16)$$

Then the displacements $\boldsymbol{u}$, $\boldsymbol{w}$ and $\boldsymbol{\phi_1}$ are approximated using Lagrangian interpolation functions:

$$u(x) \approx \sum_{j=1}^{m} u_j^e \psi_j^e(x) \; ; \; \mathrm{w}(x) \approx \sum_{j=1}^{n} w_j^e \psi_j^e(x) \; ; \; \phi_1(x) \approx \sum_{j=1}^{n} \phi_{1j}^e \psi_j^e(x) \quad (17)$$

2.3 Improved First Shear Deformation Theory

The Improved First Shear Deformation Theory (IFSDT) has the following displacement field:

$$\begin{aligned} v_1 &= u(x_1) + \phi_1(x_1).x_3 \\ v_2 &= 0 \\ v_3 &= w(x_1) + \phi_3(x_1).x_3 + \psi_3(x_1).x_3^2 \end{aligned} \qquad (18)$$

Where $\boldsymbol{u(x_1)}$ and $\boldsymbol{w(x_1)}$ are axial and transverse displacement respectively, $\boldsymbol{\phi_1(x_1)}$ and $\boldsymbol{\phi_3(x_1)}$ are the axial and transverse rotation of the cross-section and $\boldsymbol{\psi_3(x_1)}$ is a quadratic deformation in the height of the cross-section of the beam.

2.3.1 Local Stress Resultants

From the Green-Lagrange strain tensor:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \left[\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right]\alpha\right) \qquad (19)$$

Using a linear approximation ($\alpha = 0$) and replacing the displacement field we obtain:

$$\varepsilon_{11} = \frac{\mathrm{du}}{\mathrm{dx_1}} + \frac{\mathrm{d}\phi_1}{\mathrm{dx_1}}.\mathrm{x}_3 \qquad (20)$$

$$\varepsilon_{33} = \phi_3 + 2\psi_3.\mathrm{x}_3 \qquad (21)$$

$$\varepsilon_{13} = \frac{1}{2}\left(\phi_1 + \frac{\mathrm{dw}}{\mathrm{dx_1}} + \frac{\mathrm{d}\phi_3}{\mathrm{dx_1}}.\mathrm{x}_3 + \frac{\mathbf{d}\boldsymbol{\psi}_3}{\mathbf{dx}_1}.\mathbf{x}_3^2\right) \qquad (22)$$

However, the quadratic term in $\boldsymbol{\varepsilon_{13}}$ is neglected. From the Principle of Virtual Works and replacing 3D Constitutive equations we obtain the following stress resultants:

$$
\begin{aligned}
N_{11} &= C_{1111}.A.\frac{du}{dx_1} + C_{1133}.A.\phi_3 \\
M_{11} &= C_{1111}.I.\frac{d\phi_1}{dx_1} + C_{1133}.I.(2\psi_3) \\
N_{33} &= C_{1133}.A.\frac{du}{dx_1} + C_{3333}.A.\phi_3 \\
M_{33} &= C_{1133}.I.\frac{d\phi_1}{dx_1} + C_{3333}.I.(2\psi_3) \\
N_{13} &= GAK_S\left(\phi_1 + \frac{dw}{dx_1}\right) \\
M_{13} &= GIK_S\left(\frac{d\phi_3}{dx_1}\right)
\end{aligned}
\tag{23}
$$

2.3.2 Non-Local Stress Resultants

From the Euler-Lagrange equations, we obtained the IFSDT equilibrium equations. However, applying the non-local parameter on every equilibrium equation will bring higher order terms that will be neglected. Therefore, the non-local stress resultants in IFSDT are defined as:

$$
\begin{aligned}
N_{11}^{nl} &= N_{11} - \mu.\frac{df_1}{dx_1} \\
M_{11}^{nl} &= M_{11} - \mu.f_3 + C_{1133}.I.(2\psi_3) \\
N_{13}^{nl} &= N_{13} - \mu.\frac{df_3}{dx_1} \qquad M_{13}^{nl} = M_{13} \\
N_{33}^{nl} &= N_{33} \qquad M_{33}^{nl} = M_{33}
\end{aligned}
\tag{24}
$$

f_1 and f_3 are the distributed axial and transverse force respectively. The non-local stress resultants are replaced in the Virtual Work Statement.

2.3.3 Finite Element Formulation

Rearranging the terms in the Virtual Work Statement we obtain:

$$
\int_{x_1} C_{1111}.A.\frac{du}{dx_1}.\frac{d\delta u}{dx_1} - \mu.\frac{df_1}{dx_1}.\frac{d\delta u}{dx_1} - f_1.\delta u + C_{1133}.A.\phi_3.\frac{d\delta u}{dx_1}\,dx_1 = 0 \tag{25}
$$

$$
\int_{x_1} C_{1111}.I.\frac{d\phi_1}{dx_1}.\frac{d\delta\phi_1}{dx_1} + GAK_S.\phi_1.\delta\phi_1 - \mu.\frac{df_3}{dx_1}.\delta\phi_1 - \mu.f_3.\frac{d\delta\phi_1}{dx_1} + GAK_S.\frac{dw}{dx_1}.\delta\phi_1 + 2.C_{1133}.I.\psi_3.\frac{d\delta\phi_1}{dx_1}\,dx_1 = 0 \tag{26}
$$

$$\int_{x_1} \mathrm{GAK_S}.\frac{\mathrm{dw}}{\mathrm{dx_1}}.\frac{\mathrm{d\delta w}}{\mathrm{dx_1}} - \mu.\frac{\mathrm{d}\phi_3}{\mathrm{dx_1}}.\frac{\mathrm{d\delta w}}{\mathrm{dx_1}} - \phi_3.\delta\mathrm{w} + \mathrm{GAK_S}.\phi_1.\frac{\mathrm{d\delta w}}{\mathrm{dx_1}}\mathrm{dx_1} = 0 \tag{27}$$

$$\int_{x_1} \mathrm{GIK_S}.\frac{\mathrm{d}\phi_3}{\mathrm{dx_1}}.\frac{\mathrm{d}\delta\phi_3}{\mathrm{dx_1}} + \mathrm{C_{3333}}.\mathrm{A}.\phi_3.\delta\phi_3 + \mathrm{C_{1133}}.\mathrm{A}.\delta\phi_3.\frac{\mathrm{du}}{\mathrm{dx_1}}\mathrm{dx_1} = 0 \tag{28}$$

$$\int_{x_1} 4.\mathrm{C_{3333}}.\mathrm{I}.\psi_3.\delta\psi_3 + 2.\mathrm{C_{1133}}.\mathrm{I}.\delta\psi_3.\frac{\mathrm{d}\phi_1}{\mathrm{dx_1}}\mathrm{dx_1} = 0 \tag{29}$$

Then the displacements $\boldsymbol{u}$, $\boldsymbol{w}$, $\boldsymbol{\phi_1}, \boldsymbol{\phi_3}$ and $\boldsymbol{\psi_3}$ are approximated using Lagrangian interpolation functions:

$$\begin{gathered} u(x) \approx \sum_{j=1}^{m} u_j^e \psi_j^e(x)\, ;\ \mathrm{w}(x) \approx \sum_{j=1}^{n} w_j^e \psi_j^e(x)\, ;\ \phi_1(x) \approx \sum_{j=1}^{n} \phi_{1j}^e \psi_j^e(x); \\ \phi_3(x) \approx \sum_{j=1}^{n} \phi_{3j}^e \psi_j^e(x); \psi_3(x) \approx \sum_{j=1}^{n} \psi_{3j}^e \psi_j^e(x) \end{gathered} \tag{30}$$

III. FUNTIONALLY GRADED MATERIALS

Functionally Graded Materials are material combinations of two materials (usually ceramics and metals) in which the properties of material 1 will change along the height until it gets to material 2. The modulus is given by a weighted average of both material modulus given by:

$$E(z) = E_c f_c + E_m f_m \tag{31}$$

Where m and c refers to metal and ceramic respectively, and f is the volume fraction (Jn N Reddy & Arciniega, 2006). By replacing, the FGM power law is defined as:

$$f_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \qquad f_m = 1 - f_c \tag{32}$$

Where n is the exponent that will represent the variation from a material to another.

IV. VALIDATION

In the following section, numerical results are used to verify the Finite Element model presented and compare them with the present IFSDT formulation. Macro-beams and Micro-beams are evaluated by varying the Eringen's non local parameter (μ) and the Functionally Graded power law index (n) as it is in the literature. For Simply Supported and Clamped-Clamped examples, the beams are modeled using a 16 element mesh with Gauss-Legendre-

Lobatto interpolation (P=4) for primary variables to evade shear locking. A P+1 quadrature interpolation is used for all elements. In the examples herein, only transverse load f_3 is applied, whereas axial uniform load f_1 is zero.

4.1 Example 1

The transverse load applied is $f_3 = 1$. Results are normalized $\bar{w} = -w \times 100 \frac{EI}{qL^4}$. For this example the following geometric and material properties were used:

$$L = 10 \quad B = 1 \quad \text{L/H=50}$$
$$E = 30 \times 10^6 \qquad \nu = 0.30$$

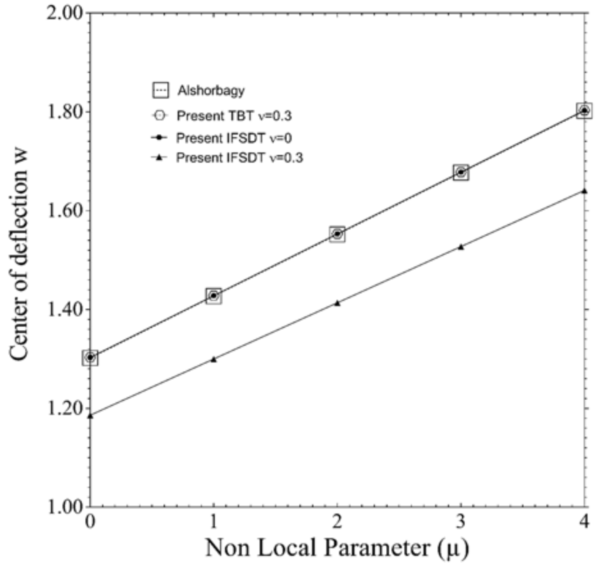

Fig. 1 Non-local Parameter vs dimensionless center of deflection for a simply supported beam under a uniformly distributed load.

Fig. 1 Non-local Parameter vs dimensionless center of deflection for a simply supported beam under a uniformly distributed load. is comparing the results shown by (Alshorbagy et al., 2013) and it is being compared with the present model. It is possible to see how the effect of the additional degrees of freedom are affected by the Poisson's modulus making it stiffer when it is increased. However, as it is shown, the Timoshenko model and IFSDT model are virtually the same in when the value of Poisson's modulus (ν=0).

4.2 Example 2

For this example an FGM beam with non-local effects is evaluated. However, the present model will only compare a normalized center of deflection in the mid-plane axis. The normalization expression and transverse load are the same as the previous example (E=Ec). The following geometric and material properties where used:

$$L = 10.00 \quad B = 1.00 \quad \text{H=1.00}$$

$$Em = 3.93 \times 10^{11} \quad \nu m = 0.2516$$

$$Ec = 7.00 \times 10^{10} \quad \nu c = 0.3462$$

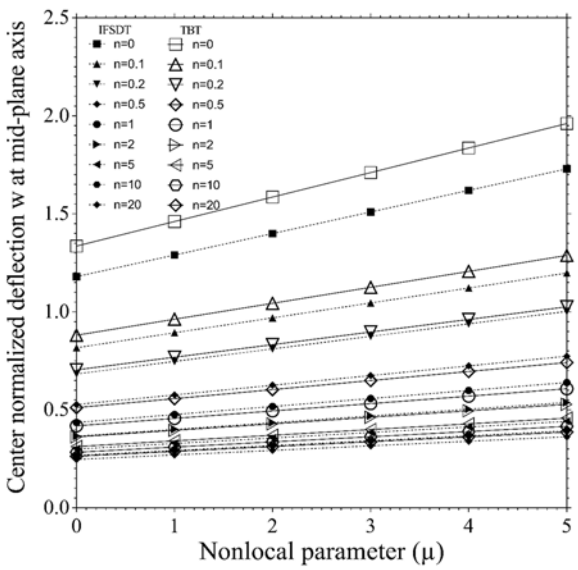

Fig. 2 Non-local Parameter vs dimensionless center of deflection at mid-plane axis for a simply supported FGM beam under a uniformly distributed load.

TBT model has been compared with (Eltaher et al., 2014) previously. Then TBT and IFSDT are compared numerically. As it was seen previously, IFSDT formulation shows less deflection when a non FGM beam (n=0 and n= ∞) is analyzed. However, in a FGM beam (n=1 for example), IFSDT shows slightly more flexible results rather than TBT.

4.3 Example 3

For this example an FGM clamped-clamped beam is evaluated under a uniformly distributed load. The model is checked with (J. N. Reddy, El-Borgi, & Romanoff, 2014) where some linear results are given as benchmark. For a unitary uniform load on a clamped-clamped homogeneous beam the deflection at x=L/2 is 1.0865×10-4 (m) using TBT and 0.9214×10-4 (m) for IFSDT. For a FGM beam of n=1 the deflection at x=L/2 is 2.5421 ×10-4 (m) using TBT and 2.1554×10-4 (m) for IFSDT. A resume of these results are given in Table 1.

Case	Reddy 2014	Present TBT	Present IFSDT
C-C (n=0)	1.0865×10-4	1.0865×10-4	0.9214×10-4
C-C (n=1)	2.5422 ×10-4	2.5421 ×10-4	2.1554×10-4

Table 1. Comparison of finite element results for the transverse center of deflection at x=L/2 for a clamped-clamped nanobeam under an unitary uniformly distributed load

V. PARAMETRIC STUDIES

For the following examples we will evaluate an FGM non-local beam under a uniformly distributed load of intensity q0=10, with multiple boundary conditions (as shown in Fig. 3 Non-local beam under uniformly distributed load using a)A cantilever Boundary Condition b)A Multi-span Beam and C) A Clamped-Hinged-Free Boundary condition with the following material and geometric properties:

$$L = 300 \quad B = 30 \quad H=15$$

$$Em = 3.00 \times 10^6 \quad \nu m = 0.30$$

$$Ec = 30.00 \times 10^6 \quad \nu c = 0.30$$

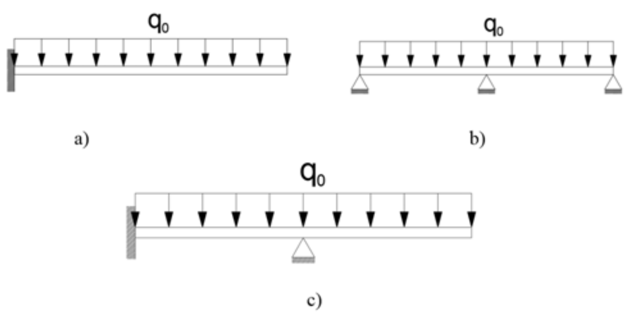

Fig. 3 Non-local beam under uniformly distributed load using a)A cantilever Boundary Condition b)A Multi-span Beam and C) A Clamped-Hinged-Free Boundary condition

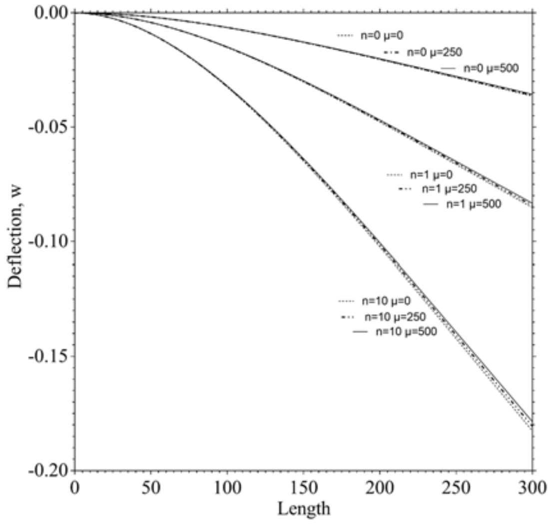

Fig. 4 Length Vs Deflection of a Non-local beam under uniformly distributed load using a cantilever Boundary Condition

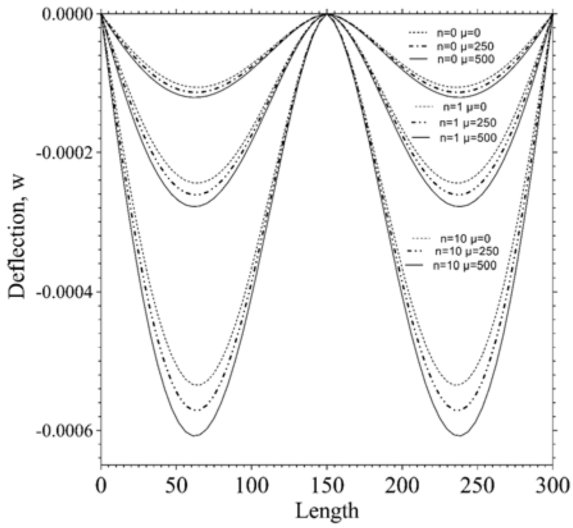

Fig. 5 Length Vs Deflection of a Non-local beam under uniformly distributed load using a multiple span boundary condition.

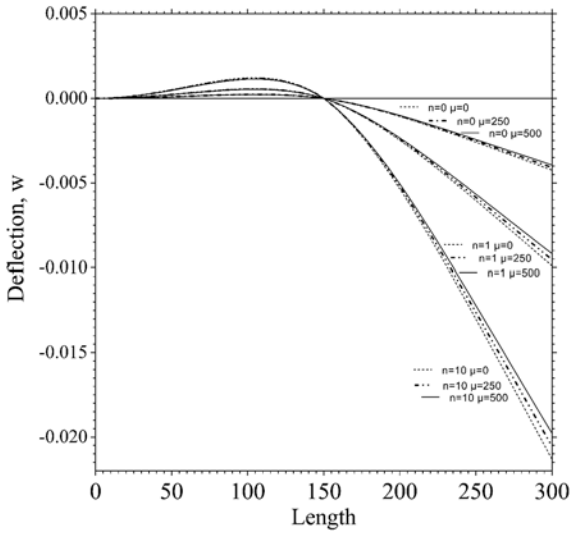

Fig. 6 Length Vs Deflection of a Non-local beam under uniformly distributed load using a Clamped-Hinged-Free boundary condition

It's convenient to remember that the deflection of a structure decreases as the stiffness increases, regardless of the boundary condition of the member in study. When non-local effect is included, the virtual work statements changes as we can see in Eq 25-29. There appear additional terms that are function of the nonlocal parameter and first derivative of transverse distributed load, which are added to conventional local forces' vector, so the stiffness matrix is the same for local and non-local approach. Thus, the new force vector has two components: local and nonlocal.

Now, it is logical to think that deflections should increase due to the increase of vector forces' values; however, this behavior is not consistent. Boundary conditions plays an important role when non-local approach is applied in structural analysis. In Fig. 4 we note that, for a cantilever beam (clamped-free beam), the deflection at the free-end decreases as the nonlocal parameter increases. Similar behavior we see in Fig. 5 for the deflection at the free-end of a clamped-hinged-free beam. In Fig. 6 it is shown the behavior according to the mentioned at the beginning of this paragraph: the deflection in the middle-span increase as the non-local parameter increase. Thus, we can say that, for simply supported boundary conditions, increase of nonlocal parameter means an increase of bending deflection, but when a totally constrained boundary condition appears, it is traduced in a decrease of deflection as nonlocal parameter is higher.

VI. CONCLUSIONS

Timoshenko and IFSDT linear non-local finite element model of FGM beams are presented with the aim of study its bending behavior. Weak form and element stiffness matrix are derived by applying Principle Virtual Statements. Non-local constitutive equations of Eringen are used to develop the non-local finite element formulation. So, after showing the results, we came to the following conclusions:

By neglecting the effect of Poisson' modulus, Timoshenko and IFSDT theories are, in fact, the same.

Non-local components of force vector do not necessarily mean an increase in deflections due to bending behavior; it depends on the boundary conditions of the beam element as those discussed in the parametric studies.

It is noticeable from Fig.4-6 that the deflection increase at the rate of increase of the power-law index "n" of FGM beams. That occurs due to the fact that for increasing values of "n", the beam material approaches that of material 2, which elasticity modulus is smaller, making the beam more flexible.

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